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The time harmonic electromagnetic scattering problem with impedance boundary conditions on a cylinder of smooth but otherwise arbitrary cross section is reduced to a pair of boundary integral equations. These integral equations are shown to have a unique square integrable solution for bounded impedances, for all real values of wave number with no exceptional values corresponding to interior resonances. This result is then employed to prove that there exists an impedance function which optimizes the amount of power scattered in an angular sector of the far field. The power in the angular sector is considered as the cost

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functional over a control set of admissable impedances consisting of a closed bounded convex set in the space dual to the space of functions integrable over the boundary. Methods for the numerical approximation of the optimal impedance are discussed.

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Scattering Control by
Impedance Loading*

by

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INTRODUCTION

In [1], the authors considered the problem of finding the optimal surface current on a cylinder which maximized the power radiated in an angular sector. That approach to antenna synthesis is further developed here in the context of a scattering problem. Specifically we prove the existence of an optimal impedance for a general cylindrical surface; optimal in the sense that when a field is incident upon the surface, the power scattered in an angular sector is maximized.

We consider an infinite cylinder of arbitrary cross-section in the presence of either $\overline{E}$ or $\overline{H}$ polarized incident fields. It is known, e.g. [8], that under suitable restrictions on the geometry and constitutive parameters of the scatterer, among which is a requirement that the radius of curvature be large relative to skin depth, the transition conditions at the surface of an imperfectly conducting scatterer may be replaced by so-called impedance boundary conditions. Then if D_+ and D_- denote the domains exterior and interior respectively to a simply connected-closed curve ∂D in R^2, the scattering problem may be reduced to finding a scalar function $u(p) = u^i(p)+u^s(p)$ such that

$$(1) \qquad (\nabla^2+k^2)u^s(p) = 0, \quad p \in D_+$$

$$(2) \qquad \frac{\partial u^s}{\partial r} - iku^s = o(1/r^{1/2})$$

$$(3) \qquad \frac{\partial u}{\partial n} + \eta(p)u = 0, \quad p \in \partial D$$

where u^i is a known incident field, $p = (x,y)$ is a point in R^2 with magnitude $r = |p| = \sqrt{x^2+y^2}$ and $\partial/\partial n$ is the derivative in the direction of the outer normal to ∂D, pointing from ∂D into D_+. Here u denotes the non-vanishing, z-component of either $\overline{E}$ or $\overline{H}$, depending on the polarization, and $\eta(p)$ denotes the equivalent surface impedance. The boundary ∂D is assumed here to be Lyapunov of order 1 (e.g. [7]) which ensures that the unit normal at p, $\hat{n}_p$, is Lipschitz continuous on ∂D.

EQUIVALENT INTEGRAL EQUATIONS

Let $R(p,q)$ denote the distance between two typical points of R^2. A fundamental solution of the Helmholtz equation will be denoted by $\gamma(p,q)$ which for convenience we normalize as

$$(4) \qquad \gamma(p,q) = -\frac{i}{2} H_0^{(1)}(kR).$$

Furthermore we let $\partial/\partial n_p^-$ and $\partial/\partial n_p^+$ denote the normal derivative when $p \to \partial D$ from D_- and D_+ respectively although the direction is always that of the outer normal.

As in [2] the single and double layer distributions at $p \in R^2$ with density $\mu \in L_2(\partial D)$ will be denoted by $(S\mu)(p)$ and $(D\mu)(p)$ respectively, i.e.,

$$(5) \qquad (S\mu)(p) := \int_{\partial D} \gamma(p,q)\mu(q)\, ds_q;$$
$$(D\mu)(p) := \int_{\partial D} \frac{\partial \gamma(p,q)}{\partial n_q}\mu(q)\, ds_q.$$

We also define, $p \in \partial D$,

$$(6) \qquad (\overline{K}\mu)(p) := (D\mu)(p).$$

Note that $\overline{K}$: $L_2(\partial D) \to L_2(\partial D)$ is compact, e.g. [7], and denote its adjoint by $\overline{K}^*$. For surface layers in R^3, the usual jump conditions hold for these densities, at least almost everywhere on ∂D and this remains true in R^2, i.e.,

$$(7) \qquad \frac{\partial}{\partial n_p^{\pm}}(S\mu)(p) = (\pm\mu + K\mu)(p),$$
$$\lim_{p \to \partial D^{\pm}} (D\mu)(p) = (\pm\mu+\overline{K}^*)(p)$$

where K is the complex conjugate of $\overline{K}$.

With this notation, representations of solutions of the Helmholtz equation obtained by applying Green's Theorem or the Helmholtz representation lead to the following representations for u^s and u^i

$$(8) \qquad \int_{\partial D}\left\{\frac{\partial u^s(q)}{\partial n_q}\gamma(p,q) - u^s(q)\frac{\partial \gamma}{\partial n_q}(p,q)\right\} ds_q = \left(S\frac{\partial u^s}{\partial n}\right)(p) - (Du^s)(p) = \begin{cases} 2u^s(p), & p \in D_+ \\ u^s(p), & p \in \partial D \\ 0, & p \in D_- \end{cases}$$

and

$$(9) \qquad (Du^i)(p) - \left(S\frac{\partial u^i}{\partial n}\right)(p) = \begin{cases} 0, & p \in D_+ \\ u^i(p), & p \in \partial D \\ 2u^i(p), & p \in D_-. \end{cases}$$

These relations may now be used to derive a pair of boundary integral equations for the total field. First note that, with (6) these relations may be written, for $p \in \partial D$, as

$$(10) \qquad u^s = S\left(\frac{\partial u^s}{\partial n}\right) - \overline{K}^*u^s$$

$$(11) \qquad u^i = \overline{K}^*u^i - S\left(\frac{\partial u^i}{\partial n}\right).$$

Consequently

$$(12) \qquad u = u^i+u^s = 2u^i + S\left(\frac{\partial u}{\partial n}\right) - \overline{K}^*u.$$

Invoking the boundary condition (3) this may be rewritten as

$$(13) \qquad (I+S\eta+\overline{K}^*)u = 2u^i,$$

where, we emphasize, u^i is the known incident field. Likewise since

$$(14) \qquad u^s(p) = \frac{1}{2}\left[S\left(\frac{\partial u^s}{\partial n}\right) - Du^s\right](p), \quad p \in D_+$$

and

$$(15) \qquad u^i(p) = \frac{1}{2}\left[Du^i - S\left(\frac{\partial u^i}{\partial n}\right)\right](p), \quad p \in D_-$$

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we have, taking normal derivatives and using the jump conditions for the derivative of a single layer,

(16) $$\frac{\partial u^s}{\partial n_p} = \frac{1}{2}\left[\frac{\partial u^s}{\partial n_p} + K\,\frac{\partial u^s}{\partial n_p}\right] - \frac{1}{2} D_n u^s$$

and

(17) $$\frac{\partial u^i}{\partial n_p} = \frac{1}{2}\left[\frac{\partial u^i}{\partial n_p} - K\,\frac{\partial u^i}{\partial n_p}\right] + \frac{1}{2} D_n u^i .$$

It follows then, with the boundary condition (3), that the total field u must satisfy the equation

(18) $$(-\eta + K\eta + D_n)u = 2\,\frac{\partial u^i}{\partial n_p} .$$

In [2] the authors have shown that in R^3 this pair of integral equations has a unique solution which gives rise to a solution (in an appropriate generalized sense) of the exterior problem. That approach may be followed in R^2 and the corresponding result is contained in the following.

Equivalence Theorem:

Let $\eta \in L_\infty(\partial D)$, $\operatorname{Im} k \geq 0$ and $\operatorname{Im}(\bar{k}\eta) \geq 0$. Then $u = u^i + u^s$ is a solution of the exterior Robin problem

(19)
i) $u \in C_2(D_+)$; $u, \frac{\partial u}{\partial n} \in L_2(\partial D)$

ii) $(\nabla^2+k^2)u^s = 0$, $p \in D_+$, $(\nabla^2+k^2)u^i = 0$, $p \in D_-$

iii) $\frac{\partial u^s}{\partial n} - iku^s = o(1/r^{1/2})$

iv) $\frac{\partial u}{\partial n_p} + \eta u = 0$ a.e. on ∂D,

if and only if u is the unique solution of

(20) $$(I + S\eta + \bar{K}^*)u = 2u^i$$

(21) $$(-\eta + K\eta + D_n)u = 2\,\frac{\partial u^i}{\partial n} .$$

THE FAR FIELD OPTIMIZATION PROBLEM

In the far field, the scattered field u^s may be written as

(22) $$u^s = \frac{e^{ikr}}{r^{1/2}} f(\theta) + o(1/r^{1/2})$$

where $f(\theta)$ is the far field coefficient. Since ∂D is bounded, we may employ the asymptotic properties of $\gamma(p,q)$ together with the integral representation (8) to represent $f(\theta)$ as

(23) $$f(\theta) = \frac{e^{-3\pi i/4}}{\sqrt{8\pi k}} \int_{\partial D} e^{-ik\hat{r}\cdot q}\Big(-\eta(q)u^s(q) - \frac{\partial u^i}{\partial n_q} - \eta(q)u^i(q) + ik\hat{r}\cdot\hat{n}_q u^s(q)\Big)\, ds_q$$

where $\hat{r} = (\cos\theta, \sin\theta)$ and $q = (x_q, y_q)$ is a point on ∂D. Defining integral operators K_1 and K_2: $L_2(\partial D) \to L_2(0,2\pi)$ in terms of the kernels $e^{-ik\hat{r}\cdot q}$ and $ik\hat{r}\cdot n_q e^{-ik\hat{r}\cdot q}$ respectively, we may write f as

(24) $$f(\theta) = K_1(\eta u^s) - K_2 u^s + K_1\left(\frac{\partial u^i}{\partial n} + \eta u^i\right).$$

Note that the far field is determined uniquely (via the unique solution of the boundary integral equations) by the impedance η.

The preliminary remarks allow us to pose a meaningful optimization problem. We consider the impedance, η, to be at our disposal and ask for those η which are optimal with respect to some criterion expressed in terms of the induced far field.

Specifically, for a given closed, bounded, convex subset U of $L_\infty(\partial D)$ called the class of admissible controls, find $\eta_0 \in U$ for which the functional

(25) $$Q_\alpha(f,\eta) = \int_0^{2\pi} \alpha(\theta)\,|f(\theta)|^2\, d\theta$$

is a maximum. Here $\alpha(\theta)$ is the characteristic function of a subset $\alpha \subset [0,2\pi]$ and Q_α represents the far field power flux through the set α, or the integral of the differential scattering cross section over the set α.

An alternate treatment of this problem in the case when k and η are real is given by Kirsch [5]. That analysis is based upon the existence of a unique solution of the exterior Robin problem proved using a layer ansatz which results in a single boundary integral equation, rather than the pair (20)-(21), where the kernel is no longer the free space Green's function but is modified as suggested by Jones [4]. The idea of using the uniqueness of solutions of the boundary integral equations to establish compactness properties of the set of admissible pairs is found in [5]. The situation here is more general and the proofs are, consequently, more complicated.

For this problem, we wish to prove the existence of an optimal choice $\eta_0 \in U$ where U is a closed bounded convex subset of $L_\infty(\partial D)$. Notice that, since $L_\infty(\partial D)$ is the dual space of $L_1(\partial D)$, U is weak* sequentially compact. Furthermore, since $L_1(\partial D)$ is separable, the relative weak*-topology on the set U is metric (see Dunford and Schwartz [3; p. 426]). Thus if $g: L_\infty(\partial D) \to X$, X a Banach space, then $g|_U$ is weak*-continuous provided $\xi_n \to \xi$ in the weak*-topology on U implies $g|_U(\xi_n) \to g|_U(\xi)$ in X. The following results show that the map $\eta \to f$ of $U \to L_2(0,2\pi)$ is continuous with respect to the weak*-topology on U. This fact, together with the continuity of the map Q_α: $L_2(0,2\pi) \to R$ will establish the required existence result.

Recall that, given any $\eta \in U$, there exists a unique solution u of the boundary integral equations (20)-(21). We will refer to such an impedance-solution pair (η, u) as an admissible pair. The set of all admissible pairs will be denoted by Ω.

__Theorem 1__: The set $\Omega \subset L_\infty(\partial D)\times L_2(\partial D)$ is __bounded__ in the product topology generated by the *norm topologies of* L_∞ *and* L_2.

__Proof__: Suppose this were not the case. Since U is bounded in $L_\infty(\partial D)$, any sequence $\{\eta_m\} \subset U$ is bounded in the L_∞ norm hence there would exist a sequence $\{(\eta_m, u_m)\} \subset \Omega$ such that $\|u_m\|_{L_2(\partial D)} \to \infty$ as $m \to \infty$. Moreover, since U is weak* sequentially compact, we may assume that $\eta_m \to \eta \in U$ in the weak*-topology of U.

Define functions $\psi_m \in L_2(\partial D)$ by

$$\psi_m := u_m/\|u_m\| .$$

Then $\|\psi_m\| = 1$ and, since $(I + \bar{K}^* + S\eta_m)u_m = 2u^i$,

(26) $$(I + \bar{K}^* + S\eta_m)\psi_m = \frac{2u^i}{\|u_m\|} .$$

But u^i is a fixed incident field and so, as $m \to \infty$, $\|2u^i/\|u_m\|\| \to 0$. Furthermore, after perhaps the extraction of a subsequence, $\psi_m \to \psi$ weakly in $L_2(\partial D)$ since $\|\psi_m\| \leq 1$ for all m. Now the operator $\bar{K}^*$ is compact on $L_2(\partial D)$ and so $\bar{K}^*\psi_m \to \bar{K}^*\psi$ strongly in $L_2(\partial D)$. Likewise, after perhaps the extraction of a further subsequence, we may assume that the sequence $\{\eta_m\psi_m\}$ is weakly convergent to a function $\rho \in L_2(\partial D)$. The compactness of the operator S now guarantees that $S(\eta_m\psi_m) \to S_\rho$ strongly and so $\psi_m \to -S_\rho - \bar{K}^*\psi$ strongly in $L_2(\partial D)$ since ψ_m satisfies equation (26). But since $\psi_m \to \psi$ weakly, we see that ψ must satisfy the homogeneous equation

$$(27)\qquad (I+\bar{K}^*)\psi+S_\rho = 0.$$

This result, together with the strong convergence of ψ_m to $\bar{K}^*\psi+S_\rho$, implies that $\psi_m \to \psi$ strongly in $L_2(\partial D)$.

On the other hand, $\psi_m \to \psi$ strongly in $L_2(\partial D)$ implies that $\eta_m\psi_m \to \eta\psi$ weakly in $L_2(\partial D)$ since, for any $\phi \in L_2(\partial D)$,

$$|\langle\eta_m\psi_m-\eta\psi,\phi\rangle| \le |\langle\eta_m(\psi_m-\psi),\phi\rangle|+|\langle(\eta_m-\eta)\psi,\phi\rangle|$$

$$(28)\qquad \le M\|\psi_m-\psi\|\|\phi\|+\left|\int_{\partial D}(\eta_m-\eta)(\psi\bar{\phi})\,ds\right|.$$

But $\psi_m \to \psi$ strongly so that the first term on the right converges to zero while the second term likewise converges to zero since $\eta_m \to \eta$ in the weak*-topology of $L_\infty(\partial D)$ and $\psi\bar{\phi} \in L_1(\partial D)$. So, in fact, $\eta_m\psi_m \to \eta\psi$ weakly, hence $S\eta_m\psi_m \to S\eta\psi$ in $L_2(\partial D)$, and the function ψ satisfies

$$(29)\qquad (I+\bar{K}^*+S\eta)\psi = 0.$$

Now, consider the sequence

$$(30)\qquad D_n\psi_m = \left(2\frac{\partial u^i}{\partial n}\right)\frac{1}{\|u_m\|} - (-\eta_m+K\eta_m)\psi_m.$$

We know from the construction of the sequence (ψ_m) that $\psi_m \to \psi$ in $L_2(\partial D)$ while $\eta_m\psi_m \to \eta\psi$ weakly in $L_2(\partial D)$. Hence the compactness of the operator K implies that the functions $D_n\psi_m$ converge weakly in $L_2(\partial D)$ to $\xi := \eta\psi-K\eta\psi$. Moreover, since ψ is a solution of (29), the results of section IV of [2] show that $\psi \in \mathcal{D}(D_n)$. We wish to show that, in fact, $D_n\psi = \xi$.

To this end, let $\phi \in C^1(\partial D)$ and note that $\phi \in \mathcal{D}(D_n^*)$ (see [6]). Look at the functional on $L_2(\partial D)$ defined by ϕ. Then we have

$$(31)\qquad \langle D_n\psi-\xi,\phi\rangle = \langle D_n\psi-D_n\psi_m,\phi\rangle+\langle D_n\psi_m-\xi,\phi\rangle.$$

The second term on the right converges to zero since $D_n\psi_m \to \xi$ weakly. The first term on the right may be rewritten as

$$(32)\qquad \langle D_n(\psi-\psi_m),\phi\rangle = \langle\psi-\psi_m,D_n^*\phi\rangle$$

which converges to zero since $\psi_m \to \psi$ strongly in $L_2(\partial D)$. Hence $D_n\psi = \xi$ and so ψ satisfies the equation

$$(33)\qquad D_n\psi = \eta\psi-K\eta\psi$$

or

$$(34)\qquad (-\eta+K\eta+D_n)\psi = 0.$$

But the pair of integral equations has a unique solution so that, again we conclude that $\psi = 0$ which is a contradiction since $\psi_n \to \psi$ in $L_2(\partial D)$ and $\|\psi_m\| = 1$. We conclude, therefore, that Ω is bounded.

<u>Theorem 2</u>: Let $L_\infty(\partial D)\times L_2(\partial D)$ be equipped with the product topology relative to the weak*-topology on $L_\infty(\partial D)$ and the norm topology on $L_2(\partial D)$. Then the set of admissible pairs is closed with respect to this product topology.

<u>Proof</u>: Here we assume that we are given a sequence of admissible pairs $\{(\eta_m,u_m)\} \subset \Omega$ such that $\eta_m \to \eta$ in the weak*-topology of $L_\infty(\partial D)$, and $u_m \to u$ strongly in $L_2(\partial D)$. We must show that $(\eta,u) \in \Omega$. We use the boundedness of Ω to ensure that there is a $\phi \in L_2(\partial D)$ such that $\eta_m u_m \to \phi$ weakly, and then use the fact that the pairs are admissible to show that the functions u_m converge strongly to $2u^i-\bar{K}^*u-S\phi$. The proof now proceeds in a manner completely analogous to the preceeding proof, and we need not repeat it here.

<u>Theorem 3</u>: The map $\eta \to f$ defined by the far field relation

$$(35)\qquad f(\theta) = \frac{e^{-3\pi i/4}}{\sqrt{8\pi k}}\int_{\partial D} e^{-ik\hat{r}\cdot q}\Big(-\eta(q)u^i(q) - \frac{\partial u^i}{\partial n_q} - \eta(q)u^i(q) + ik\hat{r}\cdot\hat{n}_q u^s(q)\Big)\,ds_q$$

where $u^s = u-u^i$ for u the solution of (20)-(21) and $\hat{r} = (\cos\theta, \sin\theta)$, is continuous from the weak*-topology of $L_\infty(\partial D)$ to the strong topology on $L_2(0,2\pi)$.

<u>Proof</u>: To see this, let (η_m) be a sequence in the closed bounded convex set $U \subset L_\infty(\partial D)$ such that $\eta_m \to \eta$ in the weak*-topology. Then to each η_m there corresponds a unique solution u_m of the pair of boundary integral equations (20)-(21). Hence the sequence of functions η_m generates a sequence of admissible pairs $\{(\eta_m,u_m)\} \subset \Omega$. Since according to Theorem 1, the class Ω is bounded there exists at least a subsequence $\{(\eta_{m_j},u_{m_j})\}$ such that $\eta_{m_j} \to \eta$ in the weak*-topology, $u_{m_j} \to u \in L_2(\partial D)$ weakly, and the sequence of products $\eta_{m_j}u_{m_j}$ converge weakly to some $\phi \in L_2(\partial D)$.

As in the proof of Theorem 1, it follows from the compactness of the operators $\bar{K}^*$ and S, that indeed the functions u_{m_j} converges strongly to the function u and so, by Theorem 2, the pair (η,u) belongs to Ω.

Returning now to the original sequence $\{u_m\}$ of solutions, we see in fact that $u_m \to u$ in $L_2(\partial D)$. Indeed, if this were not the case, then we could consider the sequence $\{\hat{u}_m\}$ consisting of all those elements of the original sequence which do not appear in the convergent subsequence $\{u_{m_j}\}$. Again we could extract a subsequence $\{\hat{u}_{m_\ell}\}$ which converges weakly to some $v \in L_2(\partial D)$. Applying the argument above to this new subsequence, we conclude that the pair (η,v) belongs to Ω. But the uniqueness of solutions of (20)-(21) for each $\eta \in L_\infty(\partial D)$ implies that $v = u$.

We have, then, that $u_m \to u$ strongly in $L_2(\partial D)$, and $\eta_m u_m \to \eta u$ weakly in $L_2(\partial D)$. Denoting the far field associated with u_m by f_m and recalling the definition of the far field (23), we see that f can be written in terms of u_m-u^i (which converges strongly to $u-u^i$) and two compact operators K_1 and K_2 which map $L_2(\partial D) \to L_2(0,2\pi)$. Specifically

$$(36)\qquad f_m(\theta) = K_1[\eta_m(u_m-u^i)]-K_2[(u_m-u^i)] + K_1\left(\frac{\partial u^i}{\partial n}+\eta_m u^i\right)$$

and so

$$(37)\qquad f_m \to K_1[\eta(u-u^i)]-K_2[(u-u^i)] + K_1\left(\frac{\partial u^i}{\partial n}+\eta u^i\right) = f$$

strongly in $L_2(0,2\pi)$.

We may now show that our optimization problem has an optimal solution in Ω.

<u>Theorem 4</u>: Let Ω be the class of admissible pairs defined above and let Q_α be defined as in (25). Then there exists a pair $(\eta_0,u_0) \in \Omega$ such that

$$(38)\qquad Q_\alpha(\eta_0,u_0) \ge Q_\alpha(\eta,u) \quad \text{for all } (\eta,u) \in \Omega.$$

<u>Proof</u>: This result follows immediately from the observation that, in light of Theorem 3, the map $\eta \to Q_\alpha(\eta,u)$ is a continuous mapping from $L_\infty(\partial D) \to L_2(0,2\pi)$ defined on the weak*-sequentially

compact set $U \subset L_\infty(\partial D)$.

Moreover from the results proven above we have the following.

Theorem 5: If $\{\eta_m, u_m\} \subset \Omega$ is a minimizing sequence such that $\eta_m \to \eta_0$ weak*. Then the unique solution u_0 of (20)-(21) associated with η_0 is the optimal total field and $u_m \to u_0$ strongly on ∂D.

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